

Addition and Subtraction to 100 Measurement and Data <u>Math in Focus</u>

Unit 4 Curriculum Guide April 29, 2019- End of School Year



ORANGE PUBLIC SCHOOLS OFFICE OF CURRICULUM AND INSTRUCTION OFFICE OF MATHEMATICS

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First Grade Unit IIII Chapter 17,11,15 Eureka Module 6 (TOPIC C,D,E,) Eureka Module 3- Topic D Eureka Module 5- Topic D

In this Unit Students will:

1.MD.3-4

- > Learn how data can be represented in a pictorial way
- Identify the value of one category and study the difference between 2 or more categories
- > Utilize tally marks to organize data

1.0A.4,8

> Determine the unknown in various locations when adding and subtracting

1.NBT.2,4,6

- > Count to 120 starting at any number less than 120
- Reade and write numerals
- > Represent number of objects with a written numeral
- > Understand the value of a digit within a number
- Compare two-digit numbers

Mathematical Practices

- > Make sense of persevere in solving them.
- > Reason abstractly and quantitatively.
- > Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- > Use appropriate mathematical tools.
- > Attend to precision.
- > Look for and make use of structure.
- > Look for and express regularity in repeated reasoning.

<u>Unit 4:</u> Addition and Subtraction to 100 Measurement & Data					
Chapter		Activity	Standard		
	Chapter Opener		1.NBT.4		
MIF	Lesson 1 : Addit	ion Without Regrouping	1.NBT.2		
Chapter 17	Lesson 2: Addit	ion With Regrouping (2 days)	1.NBT.2,4		
Addition/ Subtraction					
to 100	Lesson 3: Subtr	cacting Without Regrouping	1.NBT.6		
			1.0A.4		
	Lesson 4: Subtr	cacting with Regrouping	1.NBT.2,4		
	Eurolog Mode	1. C. Addition and Subtraction to 1	00		
	<u>Eureka moau</u>	te o: Addition and Subtraction to 1	<u>00</u>		
Торіс	Lesson	Student Lesson Objective/ Supportive	e Videos		
	Lesson 11	Add a multiple of 10 to any two-digit nu	mber within 100.		
Topic C:		https://www.youtube.com/watch?v			
Addition to	Lesson 12	Add a pair of two-digit numbers when the	ne ones digits have a		
Place Value					
Understanding	Understanding <u>https://www.youtube.com/watch?v</u>				
	Lesson 13 & 14	Add a pair of two-digit numbers when the sum greater than 10 using decomposition	ne ones digits have a on.		
		https://www.youtube.com/watch?v			
		https://www.youtube.com/watch?v			
	Lesson 15	Add a pair of two- digit numbers when t	he ones digits have		
	a sum greater than 10 with drawing. Record the total below.				
	Lesson 16- 17 Add a pair of two- digit numbers when the ones digits have a sum greater than 10 with drawing. Record the total below.				
Torio Di	Longer 10	Add a pair of two-digit numbers with va	ried sums in the		
Varied Place Value	Lesson 18 ones, and compare the results of different recording methods.				

Strategies for Addition to 100	Lesson 19	Solve and share strategies for adding two-digit numbers with varied sums.

MIF Chapter		Activity	Standard	
MIF	MIF Chapter Opener			
Chapter 11	Lesson 1: Simple Picture Graphs (2 Days)		1.MD.4	
Picture			1.OA.8	
Graphs & Bar	Lesson 2: More Picture Graphs (2 Days)		1.MD.4 1.OA.8	
Graphs	Lesson 3. Tally Charts and Bar Graphs		1.MD.4	
		Charto and Dar Grapho	1.OA.8	
	<u>Eureka Mo</u>	dule 3 Topic D: Data Interp	pretation	
Торіс	Lesson	Student Lesson Objective/ S	Supportive Videos	
Topic D:	Lesson	Collect, sort, and organize dat	a; then ask and answer	
Data	10-11	questions about the number of data points.		
Interpretation	Lesson	Ask and answer varied word problem types about a data set		
	12-13	with three categories.		
Eure	ka Module 5 1	Topic D: Application of Hal	ves to Tell Time	
	_	Student Lesson Objective / S	Supporting Videog	
Торіс	Lesson	Student Lesson Objective/ S	supportive videos	
Topic D:	Lesson 10	Construct a paper clock by	partitioning a circle and tell	
Application of		time to the hour.		
Halves to Tell	Lesson 11	Recognize halves within a circular clock face and tell		
Time		time to the half-hour.		
	Lesson 12	Recognize halves within a c	ircular clock face and tell	
		time to the half-hour.		
	Lesson 13	Recognize halves within a c	ircular clock face and tell	
		time to the half-hour.		

MIF Chapter	Activity	Standard
MIF	Lesson 2: Telling Time to the Hour	1.MD.3
Chapter 15	Lesson 3: Telling Time to the Half Hour	1.MD.3
	Problem Solving/ Chapter Wrap Up	1.OA.3

New Jersey Student Learning Standards: Operations and Algebraic Thinking

1.OA.4

Understand subtraction as an unknown-addend problem

First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems.

For Sums to 10

*<u>Think-Addition:</u>

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, "What goes with this part to make the total?" The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

For example, when working with the problem $9 - 5 = \Box$, First Graders think "Five and what makes nine?", rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what's left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem.

Example: $10 - 2 = \square$ Student: "2 and what make 10? I know that 8 and 2 make 10. So, 10 - 2 = 8."

For Sums Greater than 10

The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with Think-Addition (described above), while other students may use other strategies described below, depending on the fact. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number.

*Build Up Through 10:

This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum.

Example: $15 - 9 = \square$ Student A: "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6. 15 0 9 = 6."

Student B: "I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters."



Back Down Through 10

This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example: $16 - 7 = \Box$ **Student A**: "I'll start with 16 and take off 6. That makes 10. I'll take one more off and that makes 9. 16 - 7 = 9."

Student B: "I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2nd ten frame. Then, I'll take one more from the first ten frame. That leaves 9 on the ten frame."





1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = -3$, $6 + 6 = -3$.			
First Graders use the 1.OA.6 to solve equation	r understanding of and strategies related to addition and subtraction as described in 1.OA.4 and ons with an unknown. Rather than symbols, the unknown symbols are boxes or pictures.			
Example: Five cookie eat?	es were on the table. I ate some cookies. Then there were 3 cookies. How many cookies did I			
Student A: What goes	s with 3 to make 5? 3 and 2 is 5. So, 2 cookies were eaten.			
Student B: Five, four	, three (holding up 1 finger for each count). 2 cookies were eaten (showing 2 fingers).			
Student C : We ended 2 <i>fingers</i>).	with 3 cookies. Therese, four, five (holding up 1 finger for each count). 2 cookies were eaten (showing			
Example: Determine	the unknown number that makes the equation true. 5 - \Box = 2			
Student : 5 minus sor	nething is the same amount as 2. Hmmm. 2 and what makes 5? 3! So, 5 minus 3 equals 2.			
Now it's true!				
	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the			
	following as special cases:			
1.NBT.2	a. 10 can be thought of as a bundle of ten ones- called a "ten"			
	c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or			
	nine tens (and 0 ones)			
First Grade students are introduced to the idea that a bundle of ten ones is called a "ten". This is known as "unitizing".				
When first grade students unitize a group of ten as a whole unit ("a ten"), they are able to count groups as though they				
were individual objects. This is a monumental shift in thinking and can often be challenging young children to consider a				
group of something as "one" when all previous experiences have been counting single objects. This is the foundation of the				
place value system an	place value system and requires time and rich experiences with concrete manipulatives to develop.			



Make sure to reinforce the concept that 4 tens is the same as 40. Students should be asked to represent both ways. The use of hide zero cards will help solidify this thinking.

A student's ability to conserve number is an important aspect of this standard. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, bead, ten-frames) to make groups of ten, rather than using only pregrouped materials (e.g., Base 10 Blocks, pre-made ben sticks) that have to be "traded" or are non-proportional (e.g., money, place value disks)

Students should explore the idea that decade numbers (e.g 10, 20, 30, 40,etc) are groups of ten with no left over ones.

It is best to make a ten with unifix cubes or other materials that students can group.

As students are representing the various amounts, it is important that an emphasis is placed on the language associated with the quantity.

	Add within 100, including adding a two-digit number and a one digit number, and adding a two-digit
	number and a multiple of 10, using concrete models or drawings and strategies based on place value,
1.NBT.4	properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a
	written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds
	tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

First Grade students use concrete materials, models, drawings and place value strategies to add within 100. They do so by being flexible with numbers as they use the base-ten system to solve problems. <u>The standard algorithm of carrying or borrowing is neither an expectation nor a focus in First Grade</u>. Students use strategies for addition and subtraction in Grades K-3. By the end of Third Grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000.

Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of

Grade 4

Example: 24 red apples and 8 green apples are on the table. How many apples are on the table?

Student A: I used ten frames. I put 24 chips on 3 ten frames. Then, I counted out 8 more chips. 6 of them filled up the third ten frame. That meant I had 2 left over. 3 tens and 2 left over. That's 32. So, there are 32 apples on the table.



Student B:

I used an open number line. I started at 24. I knew that I needed 6 more jumps to get to 30. So, I broke apart 8 into 6 and 2. I took 6 jumps to land on 30 and then 2 more. I landed on 32. So, there are 32 apples on the table.



Student C:

I turned 8 into 10 by adding 2 because it's easier to add.

So, 24 and ten more is 34.

But, since I added 2 extra, I had to take them off again.

34 minus 2 is 32. There are 32 apples on the table.

Include problems that provide a context for addition as often as possible (Table 1) Equations should be written both horizontally and vertically.

Encourage students to make estimates before adding to determine if their answers are reasonable. Pose questions that require students to think about the strategies the are using to add, making connections to place value



First Graders use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g. 30, 40, 50). These opportunities develop fluency of additions and subtraction and reinforce couting up and back by 10s.

Ask students to look for patterns and explain their work

Example: There are 60 students in the gym. 30 students leave. How many students are still in the gym?

<u>Student A</u>

I used a number line. I started at 60 and moved back 3 jumps of 10 and landed on 30. There are 30 students left.



<u>Student B</u>

I used ten frames. I had 6 ten frames- that's 60. I removed three ten frames because 30 students left the gym. There are 30 students left in the gym.



Tell and write time in hours and half-hours using analog and digital clocks.

1.MD.3

For young children, reading a clock can be a difficult skill to learn. In particular, they must understand the differences between the two hands on the clock and the functions of these hands. By carefully watching and talking about a clock with only the hour hand, First Graders notice when the hour hand is directly pointing at a number, or when it is slightly

ahead/behind a number. In addition, using language, such as "about 5 o'clock" and "a little bit past 6 o'clock", and "almost 8 o'clock" helps children begin to read an hour clock with some accuracy. Through rich experiences, First Grade students read both analog (numbers and hands) and digital clocks, orally tell the time, and write the time to the hour and half-hour.



All of these clocks indicte the hour of "two", although they look slightly different.

This is an important idea for students as they learn to tell time.

Organize, represent and interpret data with up to three categories; ask and answer questions about1.MD.4the total number of data points, how many in each category and how many more or less are in one
category than in another

Students can use graphs and charts to organize and represent data about things in their lives (e.g., favorite colors, pets, shoe types, and so on).

Charts may be constructed by groups of students as well as by individual students. These activities will help prepare students for work in grade two when they draw picture graphs and bar graph.

Ensure students limit categories to only three choices

When students collect, represent, and interpret data, they reinforce number sense and counting skills. When students ask and answer questions about information in charts or graphs, they sort and compare data. Students use addition and subtraction and comparative language and symbols to interpret graphs and charts

Require students to write 2 sentences about their graphs including analysis of the most and least in a category and a third sentence to tell how many more or less are in one category than another.

This standard is designed to work well with the compare situations (see Table 1)

Data can be collected and organized, graphed, and/or displayed throughout daily activities such as calendar, weather, attendance, question of the day.

M : Major Content

S: Supporting Content

A : Additional Content

Math In Focus Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
	Chapter Opener	Teacher Materials	Recall Prior Knowledge (RPK) can take place just
	Assessing Prior Knowledge	Quick Check	before the pre-tests are given and can take 1-2
		Pre-Test (Assessment	days to front load prerequisite understanding
		Book)	
	The Pre Test serves as a	Recall Prior Knowledge	Quick Check can be done in concert with the
	diagnostic test of readiness of		RPK and used to repair student
L.	the upcoming chapter	Student Materials	misunderstandings and vocabulary prior to the
TE		Student Book (Quick	pre-test ; Students write Quick Check answers
RE		Check); Copy of the Pre	on a separate sheet of paper
4		Test; Recall prior	
		Knowledge	Quick Check and the Pre Test can be done in
			the same block (See Anecdotal Checklist; Transition
			Guide)
			Posall Prior Knowledge - Quick Check - Pro Test
			Recall Filor Kilowieuge – Quick Check – Fie Test
	Direct	Teacher Edition	The Warm Up activates prior knowledge for
	Involvement/Engagement	5-minute warm up	each new lesson
5	Teach/Learn	Teach; Anchor Task	Student Books are CLOSED; Big Book is used in
JEV			Gr. K
Se	Students are directly involved	Technology	Teacher led; Whole group
BAG	in making sense, themselves,	Digi	Students use concrete manipulatives to explore
Ň	of the concepts – by		concepts
E .	interacting the tools,	Other	A few select parts of the task are explicitly
RE	manipulatives, each other,	Fluency Practice	shown, but the majority is addressed through
ā	and the questions		the hands-on, constructivist approach and
			questioning
			Teacher facilitates; Students find the solution
\square	Guided Learning and Practice	Teacher Edition	Students-already in pairs /small, homogenous
Y	Guided Learning	Learn	ability groups; Teacher circulates between
			groups; Teacher, anecdotally, captures student
		Technology	thinking
U J		Digi	
Z		Student Dook	Small Crown w/Teacher size lating among
AR		Student Book	small Group w/ reacher circulating among
LE O		Hands-on Activity	Bruisit Concrete and Model Drawing: Peteach
DED		Hallus-off Activity	Tooshor sponds majority of time with struggling
10			learners: some time with on level and less time
0			with advanced groups
			Games and Activities can be done at this time
			Sumes and Activities can be done at this time

INDEPENDENT PRACTICE	Independent Practice <i>A formal formative</i> <i>assessment</i>	Teacher Edition Let's Practice Student Book Let's Practice Differentiation Options All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	Let's Practice determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives CAN be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
RACTICE	Extending the Lesson	Math Journal Problem of the Lesson Interactivities Games	
ADDITIONAL P	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POSTTEST	End of Chapter Wrap Up and Post Test	Teacher Edition Chapter Review/Test Put on Your Thinking Cap Student Workbook Put on Your Thinking Cap Assessment Book Test Prep	 Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed Individually (e.g. for homework) then reviewed in class As a 'mock test' done in class and doesn't count As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own

Misconceptions:

- Students may add incorrectly when counting on because they lose track of how many numbers they have counted.
- Students may forget to record the regrouped ten when adding or subtracting.
- Students may record the regrouped ten with the ones digit, but forget to cross out and record the new tens digit.
- Students may have difficulty completing the statements that require comparing the numbers in the graph.
- Students may have trouble making the connection between the tally marks and the bars on the bar graph. Have them count each individual mark.
- Because the hour hand is between two numbers, children may be confused about which hour to record. Remind them that they are looking for a time past the hour, not before the hour.

PARCC Assessment Evidence/Clarification Statements				
CCSS	Evidence Statement	Clarification	Math Practices	
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8+?=11, 5=?-3, 6+6=?.	i) Interviews (individual or small group) should target students' thinking strategies for determining the unknown in an addition or subtraction equation relating 3 whole numbers. Thinking strategies expected in Grade 1 (Level 2 and 3) are defined in 1.OA.6 and in OA Progression	MP 7,8	
1.NBT.2- 1	Understand that the two digits of a two-digit number represent amounts of tens and ones.	 i)Tasks should focus on the understanding of two-digit numbers as some number of "tens" and some number of "ones." ii) Interviews (individual or small group) should target this understanding 	MP 7,8	
1.NBT.2- 2	Understand that 10 can be thought of as a bundle of ten ones — called a "ten.".	 i)Tasks should focus on the understanding of ten "ones" as a unit of one "ten." ii) Interviews (individual or small group) should target this understanding. 	MP 7,8	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students both how a problem was solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.









Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.1.OA.6 Add and Subtract within 10.2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.0A.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 4 = 13 3 1 = 10 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- o place value,
- \circ properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- <u>Conceptual understanding</u>: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- <u>Strategic competence</u>: ability to formulate, represent, and solve mathematical problems;
- <u>Adaptive reasoning</u>: capacity for logical thought, reflection, explanation, and justification;
- <u>Productive disposition</u>: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.



(Wiliam 2007, pp. 1054; 1091)

Unit 4 Assessment / Authentic Assessment Framework				
Assessment	CCSS	Estimated Time	Format	
Chapter 17				
Optional Chapter 17 Test	1.NBT.2, 4, 6 1.OA.4	1 block	Individual	
Chapter 11				
Optional Chapter 11 Test	1.OA.8 1.MD.4	1 block	Individual	
Chapter 15				
Optional Chapter 15 Test	1.OA.3 1.MD.3	1 block	Individual	

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

- 1. I can solve problems without giving up.
- 2. I can think about numbers in many ways.
- 3. I can explain my thinking and try to understand others.
- 4. I can show my work in many ways.
- 5. I can use math tools and tell why I choose them.
- 6. I can work carefully and check my work.
- 7. I can use what I know to solve new problems.
- 8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Make sense of problems and persevere in solving them

Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, "Does this make sense?" First Grade students' conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences..

2 Reason abstractly and quantitatively

1

Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: "There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?" first grade students may write 20 + 40 = 60 to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing 20 + 10 + 10 + 10 = 60. The numbers and equations written illustrate the students' thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

Construct viable arguments and critique the reasoning of others

Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.

Model with mathematics

A Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.

Use appropriate tools strategically

Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher order thinking skills are developed

Attend to precision

6

Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (=, >, 3, a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."

Look for and make use of structure

Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation)

Look for and express regularity in repeated reasoning

Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving 8+7+2, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (6+6 = 12), 7 of one color and 5 of another (7+5 = 12), etc. In essence, students repeatedly find numbers that add up to 12.

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions						
Practice	Description/ Questions					
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task?					
	How do you respond to the work that students are likely to produce?					
	Which strategies from student work will be most useful in addressing the mathematical goals?					
2. Monitoring	Paying attention to what and how students are thinking during the lesson.					
	Students working in pairs or groups					
	Listening to and making note of what students are discussing and the strategies they are using					
	Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)					
3. Selecting	This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.					
4. Sequencing	What order will the solutions be shared with the class?					
5. Connecting	Asking the questions that will make the mathematics explicit and understandable.					
	Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.					



Note:

- Place emphasis on the flow of the lesson in order to ensure the development of students' conceptual understanding.
- Outline each essential component within lesson plans.
- Math Workstations may be conducted in the beginning of the block in order to utilize additional support staff.
- Recommended: 5-10 technology devices for use within **TECHNOLOGY** and **FLUENCY** workstations.

First Grade PLD Rubric

Got It		Not There Yet			
Evidence shows that the student essentially has		Student shows evidence of a major misunderstanding, incorrect concepts			
the target concept or big	g math idea.	or procedure, or a failur	to engage in the task.		
PLD Level 5: 100%	PLD Level 4: 89%	PLD Level 3: 79%	PLD Level 2: 69%	PLD Level 1: 59%	
Distinguished	Strong Command	Moderate Command	Partial Command	Little Command	
command					
Student work shows	Student work shows	Student work shows	Student work shows	Student work shows	
distinguished levels of	strong levels of	moderate levels of	of the mathematics	the methometics	
mathematics	mathematics	mathematics	of the mathematics.	the mathematics.	
mathematics.	mathematics.	mathematics.	Student constructs	Student attemnts to	
Student constructs	Student constructs	Student constructs and	and communicates an	constructs and	
and communicates a	and communicates a	communicates a	incomplete response	communicates a	
complete response	complete response	complete response	based on student's	response using the:	
based on	based on	based on	attempts of	Tools:	
explanations/reasonin	explanations/reasoni	explanations/reasonin	explanations/	 Manipula 	
g using the:	ng using the:	g using the:	reasoning using the:	tives	
Tools:	Tools:	Tools:	Tools:	o Five	
 Manipula 	 Manipula 	 Manipulat 	o Manipula	Frame	
tives	tives	ives	tives	o Ten	
◦ Five	o Five	○ Five	○ Five	Frame	
Frame	Frame	Frame	Frame	• Number	
o Ten	o Ten	o Ten	o Ten	Line Dout Dout	
- Number	Frame Number	Frame Number	- Number	o Part-Part-	
0 Nulliber	0 Nulliber	0 Nulliber	0 Nulliber	Model	
\circ Part-Part-	\circ Part-	\circ Part-Part-	\circ Part-Part-	Strategies	
Whole	Part-	Whole	Whole	\circ Drawings	
Model	Whole	Model	Model	\circ Counting	
Strategies:	Model	Strategies:	• Strategies:	All	
o Drawings	• Strategies:	o Drawings	 ○ Drawings 	 Count 	
 Counting 	 Drawings 	 Counting 	 Counting 	On/Back	
All	 Counting 	All	All	o Skip	
 Count 	All	o Count	 Count 	Counting	
On/Back	 Count 	On/Back	On/Back	 Making 	
o Skip	On/Back	o Skip	o Skip	Ten	
Counting	o Skip	Counting	Counting	 Decompo 	
о Making	Counting Malving	o Making	o Making	Sing	
				Precise use of	
sing	\circ Decompo	ing	sing	math vocabulary	
Number	sing	Number	Number	matir votabulary	
Precise use of	Number	Precise use of	Precise use of	Response includes	
math vocabulary	Precise use of	math vocabulary	math vocabulary	limited evidence of the	
Response includes an	math vocabulary	j i i i i i i i i i j	, , , , , , , , , , , , , , , , , , ,	progression of	
efficient and logical		Response includes a	Response includes an	mathematical	
progression of	Response includes a	logical but incomplete	incomplete or illogical	reasoning and	
mathematical	logical progression of	progression of	progression of	understanding.	
reasoning and	mathematical	mathematical	mathematical		
understanding.	reasoning and	reasoning and	reasoning and		
	understanding.	understanding.	understanding.		
P		Contains minor errors.	.		
5 points	4 points	3 points	2 points	1 point	

Ms. Chatlos is buying school supplies for her 1st grade class.

PART A

Ms. Chatlos buys 47 blue pens and 40 black pens. How many pens does she buy in all?

PART B

Ms. Chatlos buys 6 small notebooks and 68 big notebooks. How many notebooks does she buy in all?

PART C

Ms. Chatlos buys 40 boxes of crayons. She returns 20 boxes. How many boxes does she have left?

PART D

Ms. Chatlos has 37 pencils. She buys 36 more. How many pencils does she have all together?

CCSS.MATH.CONTENT.1.NBT.C.4

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

CCSS.MATH.CONTENT.1.NBT.C.6

Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Answers:

Part A: 87 pens

Part B: 74 notebooks

Part C: 20 boxes

Part D: 73 pencils

No Command	Partial Accomplishment	Substantial Accomplishment	Complete Mastery
All is incorrect	Students who demonstrate	Students who demonstrate	Students who demonstrate
	partial accomplishment	substantial accomplishment	complete mastery
	accurately answer 1 part.	accurately answer 2-3 parts.	accurately answer all 4
			parts.

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **<u>21st Century Career Ready Practices</u>**.

Resources

Think Central: https://www-k6.thinkcentral.com/ePC/start.do

Engage NY http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools http://commoncoretools.me/ http://www.ccsstoolbox.com/ http://www.achievethecore.org/steal-these-tools

Achieve the Core http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives

http://nlvm.usu.edu/en/nav/vlibrary.html http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000 http://www.thinkingblocks.com/

Illustrative Math Project : http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: <u>http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/</u>

Georgia Department of Education:<u>https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx</u> Gates Foundations Tasks:<u>http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf</u>

Minnesota STEM Teachers' Center: <u>http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships</u>

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com





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TR5

Counters and Numerals (13-16)



Counters and Numerals (17-20)



Counters and Numerals (21-24)



Counters and Numerals (25-28)



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